

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9610

Roll No.

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B.Tech.**(SEMESTER-II) THEORY EXAMINATION, 2011-12****MATHEMATICS – II****Time : 3 Hours]****[Total Marks : 100**

Note : Attempt questions from each Section as indicated. The symbols have their usual meaning.

Section – A

1. Attempt **all** parts of this question. Each part carries **2** marks.

10 × 2 = 20

- (a) Prove that if M and N in

$$M(x, y)dx + N(x, y) dy = 0$$

Satisfy the equation

$$\frac{\partial M}{\partial y} + \frac{3}{y} M = \frac{\partial N}{\partial x},$$

then y^3 is an integrating factor.

- (b) $y = (c_1 + c_2x + c_3x^2)e^x$ is the solution of the differential equation, c_1, c_2, c_3 are constants.

- (c) Classify the singular points of the differential equation

$$x^2 \frac{d^2y}{dx^2} + ax \frac{dy}{dx} + by = 0,$$

a and b are constants.

- (d) Show that $\sum_{n=0}^{\infty} P_n(x) = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1-x}}$.

- (e) State the conditions for the existence of Laplace Transform.

- (f) State Convolution Theorem.

- (g) State Dirichlet conditions for the expansion of $f(x)$ in Fourier series.

- (h) Classify the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + 3 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$$

- (i) Explain briefly the method of separation of variables in solving a given partial differential equation.

- (j) The two-dimensional wave equation is _____.

Section – B

Attempt any **three** parts of this question. Each part carries equal marks.

3 × 10 = 30

2. (a) An R–L–C circuit connected in series has $R = 90$ Ohms, $C = \frac{1}{140}$ farad, $L = 10$ henries and an applied voltage $E(t) = 10 \cos t$. Assuming **no initial charge** on the capacitor, but an initial current of 1 ampere at $t = 0$, when the voltage is first applied, find the subsequent charge on the capacitor and the amplitude of the steady-state charge.
- (b) Find the series solution, about $x = 0$, of the equation $x \frac{d^2y}{dx^2} + \frac{dy}{dx} - xy = 0$, by the Frobenius method.
- (c) Using convolution, solve the initial value problem

$$\frac{d^2y}{dt^2} + 9y = \sin 3t$$

 given $y = 0, \frac{dy}{dt} = 0$ at $t = 0$.
- (d) Find the Fourier series representation upto second harmonics of $f(x)$ which is given in the following table :
- | | | | | | | |
|--------|---|----|----|----|----|----|
| x | 0 | 1 | 2 | 3 | 4 | 5 |
| $f(x)$ | 9 | 18 | 24 | 28 | 26 | 20 |
- (e) Solve the boundary value problem

$$\frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial y^2} = 0,$$

 satisfying the conditions $z(x, 0) = 0, z(x, \pi) = 0, z(0, y) = 4 \sin 3y$.

Section – C

Attempt any **two** parts from each question of this Section. Each part carries equal marks.

5 × 10 = 50

3. (a) If $(x + y)^n$ is an integrating factor of $(4x^2 + 2xy + 6y)dx + (2x^2 + 9y + 3x)dy = 0$, find n and then solve the equation.
- (b) Solve : $(D^2 - 3D + 2)y = \sin(e^{-x})$
- (c) Solve : $\left(\frac{d}{dx} + \frac{1}{x}\right)^2 y = x^{-4}$.
4. (a) Prove that $(n + 1) P_{n+1}(x) = (2n + 1)x P_n(x) - nP_{n-1}(x)$.
- (b) Show that

$$\int x J_0^2(x) dx = \frac{1}{2} x^2 \left[J_0^2(x) + J_1^2(x) \right] + c.$$
- (c) Show that

$$J_3(x) = \left(\frac{8}{x^2} - 1 \right) J_1(x) - \frac{4}{x} J_0(x)$$

5. (a) State second shifting theorem for Laplace transform and hence find the Laplace transform of the following function :

$$f(t) = \begin{cases} e^{t-a}, & t > a \\ 0, & t < a \end{cases}$$

- (b) Find the inverse Laplace transform of $\frac{s}{(s^2 + a^2)^3}$.

- (c) Show that

$$\int_0^{\infty} \frac{\cos 6t - \cos 4t}{t} dt = \log \frac{2}{3}$$

6. (a) Find the Fourier series of $f(x) = \frac{3x^2 - 6\pi x + 2\pi^2}{12}$ in the interval $(0, 2\pi)$. Hence,

$$\text{deduce that } \frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$$

- (b) Find the half-range sine series of $f(x) = (lx - x^2)$ in the interval $(0, l)$. Hence, deduce that

$$\frac{\pi^3}{32} = 1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots$$

- (c) Solve : $(D^2 + DD' - 6D'^2)z = y \sin x$

7. (a) Solve : $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$,

α constant, subject to the boundary conditions $u(0, t) = 0$, $u(\pi, t) = 0$ and the initial condition $u(x, 0) = \sin 2x$.

- (b) Find the deflection of the vibrating string which is fixed at the ends $x = 0$ and $x = 2$ and the motion is started by displacing the string into the form $\sin^3\left(\frac{\pi x}{2}\right)$ and releasing it with zero initial velocity at $t = 0$.

- (c) Determine the electromotive force $e(x, t)$ in a transmission line of length b , t seconds after the ends were suddenly grounded. Assume that R and G are negligible and the initial conditions are $i(x, 0) = i_0$ and $e(x, 0) = e_1 \sin \frac{\pi x}{b} + e_5 \sin \frac{5\pi x}{b}$. Here G and R denote the terms for the effect of leakage and resistance respectively.